Olimpiada Naţională de Matematică
- etapa locală – 11 februarie 2012
Clasa a VI-a
Soluţii şi barem

Varianta 2

1. Fie $(a;b)=d \Rightarrow$ a = dx unde $(x;y)=1$, b = dy

Obţinem $[a;b]=dxy$.

Din $[a;b]\cdot(a;b)=864 \Rightarrow d^2xy=864$, ... 1p

Din $[a;b]+(a;b)=84 \Rightarrow dxy+d=84 \Rightarrow d(xy+1)=84, \Rightarrow$ 1p

$\Rightarrow d \in D_{84}=\{1,2,3,4,6,7,12,14,21,28,42,84\}$ 1p

Obţinem tabelul: 1p

d	1	2	3	4	6	7	12	14	21	28	42	84
xy	83	41	27	20	13	11	6	5	3	2	1	0

Deoarece $d^2xy=864 \Rightarrow d=12$ si $xy=6$, ... 1p.

Obţinem tabelele:

x	1	2	3	6
y	6	3	2	1

.... 1p

a	12	24	36	72
b	72	36	24	12

.... 1p

2. $n = 7c_1 + 1 \;\big|\, +20 \Rightarrow n + 20 = 7c_1 + 21 = M_1$ 1p

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$\Rightarrow n = [7, 8, 9] = 504 \Rightarrow n + 20 = 504k$ 1p

Dacă $k = 1 \Rightarrow n + 20 = 504 \Rightarrow n = 484$ 1p

$k = 2 \Rightarrow n + 20 = 1008 \Rightarrow n = 988$ 1p

$k \geq 3 \Rightarrow$ numerele sunt mai mari de trei cifre. 1p

3. a) $\overline{0,1(a)}+\overline{0,(a)}+\overline{0,a(1)} = \frac{\overline{1a}-1}{90}+\frac{a}{9}+\frac{\overline{a1}-a}{90} =$ 1p

$\frac{10+a-1}{90}+{}^{10/}\frac{a}{9}+\frac{10a+1-a}{90}=\frac{a+9+10a+9a+1}{90}=$ 1p

$= \dfrac{20a+10}{90} = \dfrac{10(2a+1)}{90} = \dfrac{2a+1}{9}$ 1p

$\dfrac{2a+1}{9} \in \mathbb{N} \Rightarrow a = 4$ 1p

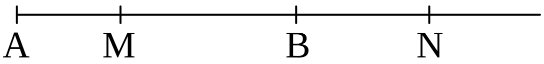
b) $\dfrac{x+1}{2} + \dfrac{x+2}{3} + ... + \dfrac{x+99}{100} = 99 \Leftrightarrow \left(\dfrac{x+1}{2} - 1\right) + \left(\dfrac{x+2}{3} - 1\right) + ... + \left(\dfrac{x+99}{100} - 1\right) = 0 \Leftrightarrow$ .. 1p

$\Leftrightarrow \dfrac{x-1}{2} + \dfrac{x-1}{3} + \dfrac{x-1}{4} + ... + \dfrac{x-1}{100} = 0 \Leftrightarrow$ 1p

$\Leftrightarrow (x-1)\left(\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + ... + \dfrac{1}{100}\right) = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$ 1p

4. a) Considerând suma minimă $S = 1+2+3+...+27 = 378 > 360 \Rightarrow$ cel puţin două sunt congruente.................. .... **3p**

b) Fie figura:



Avem MA = kMB şi NB = kAB = k(AM + MB) = kAM + kMB = k^2MB + kMB = = kMB(k + 1).

Cum AN = AM + MB + BN $\Rightarrow$ AN = kMB + MB + kMB(k + 1) = MB(k + 1) + + kMB(k + 1) = MB(k + 1)(1 + k) = MB$(k + 1)^2$ $\Rightarrow \dfrac{AN}{MB} = (k+1)^2$ ……..................... **4p**